



FEEVER Meeting – October 13, 2014 ■ ■

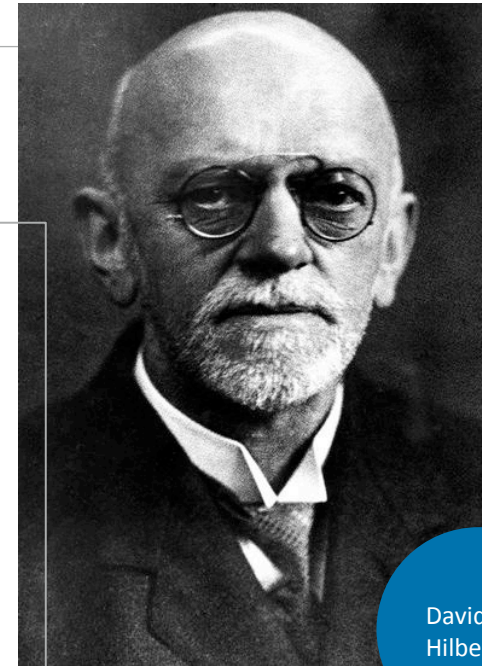
- **Type Inference in Multirate Faust Is Undecidable**

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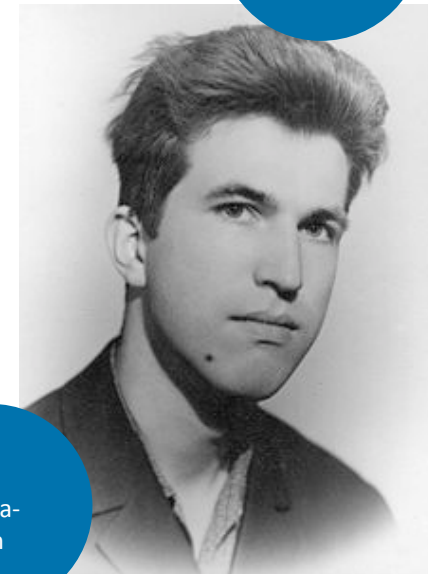
# Hilbert's Problems

- 23 key problems for the 20th century (Paris, 1900)
- Problem examples:
  - 1<sup>st</sup> – Prove the Continuum Hypothesis. (?)
  - 13<sup>th</sup> – Solve 7-th degree equation using continuous functions of two parameters(~)
  - 17<sup>th</sup> – Express a nonnegative rational function as quotient of sums of squares (P)
- 10<sup>th</sup> problem:  
Find an algorithm to determine whether a given polynomial Diophantine equation with integer coefficients has an integer.
- The 10th problem is undecidable (Matiyasevich, 1970)



David Hilbert

(1)



Yuri Matiyasevich

(2)

# Type Inference as Hilbert's 10<sup>th</sup> Problem

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- The Multirate Faust Type Inference Problem:

*Given a Faust expression  $E$ , decide whether there exists a type for  $E$  consistent with Orlarey-Jouvelot's Faust multirate static semantics.*

- Theorem (UFTI): *Multirate Faust type inference is undecidable.*
- Idea: Encode the type inference problem as a Hilbert's 10<sup>th</sup> problem.
- Diophantine equations (DE):
  - $ax+by = 1$
  - $x^n+y^n = z^n$  (Théorème de Fermat)
  - $2x^2y^3+4x^5+7 = 0$
- Finding an algorithm for solving DEs is impossible (undecidable)

# Faust Encoding (No Subtyping)

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- Faust Vector API :
  - New vectors, via `vectorize` :  $(t^f, \text{int}[n,n]^f) \rightarrow (\text{vector}_n(t)^f)$
  - Vector access, via `[]` :  $(\text{vector}_n(t)^f, \text{int}[0,n-1]^f) \rightarrow (t^f)$

- How to create unknowns  $(x, y)$ ?

```
constantize = 0, _ : vectorize : !;
```

```
new_unknown = _ <: constantize, _ ;
```

- Remarks:
  - No output for `constantize`, but enforces inputs of type  $b[n,n]$
  - Connector `<:` typing rule enforces equality between intermediate signals

# Faust Encoding (No Subtyping) (contd.)

- How to create monomials ( $x^n$ ) ?

```
power(x,0) = 1;  
power(x,1) = x;  
power(x,n) = x <: _,power(_,n-1) : *;
```

- Remarks:

- No duplication of  $x$  authorized (we need to use  $\_$  as arguments)

- How to encode polynomials ( $x^3y$ )? Case of  $p(x,y,z) = 2x^3y+yz^4+xyz+5$ :

```
p(x,y,z) =  
  (x,y,z) <:  
    (power(_,3),_ ,! : *),  
    (!,_,power(_,4) : *),  
    ((_,_ : *),_ : *),  
    5  
  :> _;
```

# Faust Encoding (No Subtyping) (contd.)

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- How to encode tests ( $p(x) = 0$ )?

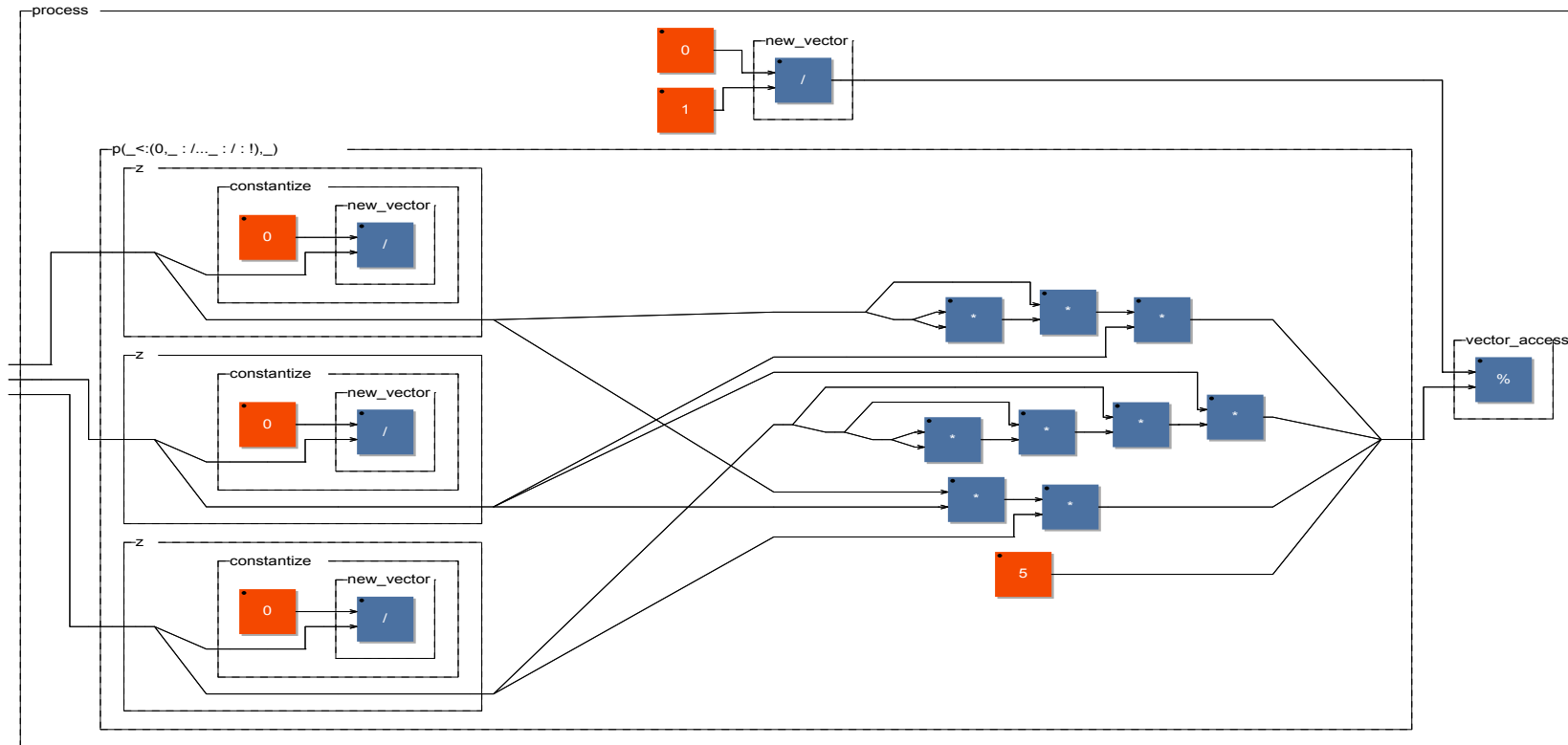
```
is_zero(p) = (0,1 : vectorize),p : [] : !;
```

- How to encode Hilbert's 10<sup>th</sup> problem?

```
x = new_unknown;  
y = new_unknown;  
z = new_unknown;
```

- process = is\_zero(p(x,y,z));

# Faust Encoding (No Subtyping) (end)



# Handling Subtyping

- Flexibility induced by the subtyping rule (interval widening)
- Loss of constant property for signals
- Subtyping can remove constancy:
  - at top level... but no output!
  - when connecting constant signals to the inputs of polymorphic (either on  $t$  or on  $[n, n']$ ) processors
- Hilbert encoding of signal processors without subtyping:
  - constant (e.g., 5)
  - constant-preserving (e.g., \*)
  - fully polymorphic (e.g., `vectorize` or `_`)
- **constant(E)** ensures that all the intermediate signals within **E** are constant:

$\text{constant}(E : E') = \text{constant}(E) <: \text{constantize}, \text{constant}(E')$

$\text{constant}(E \text{ op } E') = \text{constant}(E) \text{ op } \text{constant}(E')$

$\text{constant}(I) = I$

- Remarks:
  - Hypothesis: connection between  $E$  and  $E'$  has arity one
  - **constant** can only be applied to scalar signals
  - Lemma: *If  $E_i$  is constant-preserving,  $\text{constant}(E_1 : E_2)$  is too.*



- Theorem UFTI grounds an intuition on the power of Faust type system
- Rates not necessary to prove undecidability
- Opportunities:
  - Language type extensions to handle user assertions
  - Other « undecidable constructs » are ok (null vectors?)
- Future work:
  - Use Faust/CoQ to formally prove the UFTI theorem
  - Do explicit input types kill the proof?



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